# शिक्षा निदेशालय, राष्ट्रीय राजधानी क्षेत्र दिल्ली

# Directorate of Education, GNCT of Delhi

**अभ्यास प्रश्न पत्र**

**Practice Paper**

**Term-II (2021-22)**

**कक्षा – XII**

**Class – XII**

**गणित (कोड: 041)**

**Mathematics (Code: 041)**

**समय**: 2 **घंटे** — **अधिकतम अंक**: 40

**Time: 2 hours** — **Maximum Marks: 40**

**सामान्य निर्देश:**

**1- इस प्रश्न पत्र मे तीन खंड हैं -अ ,ब, स प्रत्येक भाग अनिवार्य है।**
**2-खंड अ में छह संक्षिप्त उत्तर -I वाले प्रश्न हैं जिसमें प्रत्येक 2 अंक का है।**
**3-खंड ब में चार संक्षिप्त उत्तर -II वाले प्रश्न हैं जिसमें प्रत्येक 3 अंक का है।**
**4 -खंड स में चार दीर्घ उत्तर वाले प्रश्न हैं जिसमें प्रत्येक 4 अंक का है।**
**5- कुछ प्रश्नों मे आंतरिक विकल्प दिया है ।**
**6-प्रश्न 14 केस पर आधारित प्रश्न है जिसके दो उपभाग हैं प्रत्येक 2 अंक का है।**

***General Instructions:***
***1. This question paper contains three sections – A, B and C. Each part is compulsory.***
***2. Section - A has 6 short answer type (SA1) questions of 2 marks each.***
***3. Section – B has 4 short answer type (SA2) questions of 3 marks each.***
***4. Section - C has 4 long answer type questions (LA) of 4 marks each.***
***5. There is an internal choice in some of the questions.***
***6. Q14 is a case-based problem having 2 sub parts of 2 marks each***

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| | **खंड - अ**<br>**SECTION – A** | |
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| **प्र. स.**<br>**Q. No.** | | **अंक**<br>**Marks** |
| **1** | ज्ञात कीजिए Evaluate $\int \left(\frac{1+x}{1+x^2}\right)dx$<br>अथवा (OR)<br>ज्ञात कीजिए Evaluate $\int \left(\frac{3x-1}{(x-2)^2}\right)dx$ | **2** |
| **2.** | अवकल समीकरण की कोटी और घात का योग ज्ञात कीजिए ।<br>Write the sum of the order and degree of differential equation<br>$\frac{d}{dx}\{\left(\frac{dy}{dx}\right)^3\}=0$ | **2** |

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| 3 | यदि दो इकाई सदिशों का योग एक इकाई सदिश है तब सिद्ध कीजिए कि उनके अन्तर का परिमाण $\sqrt{3}$ है। <br> If the sum of two unit vectors is unit vector , prove that magnitude of their difference is $\sqrt{3}$ | 2 |
| 4 | दी हुई रेखा की दिककोज्याएं ज्ञात कीजिए . <br> **Find the direction cosines of the line** <br> $\frac{x+2}{4} = \frac{3x-y}{2} = \frac{2z-5}{3}$ | 2 |
| 5 | एक थैले जिसमे 5 सफेद तथा 4 हरे रंग की गेंदें हैं इनमें से 3 गेंदें एक एक करके निकाली जाती हैं तथा पहली गेंद दूसरे के निकालने से पहले वापिस नहीं रखी जाती। हरी गेंदों की संख्या का प्रायिकता बंटन ज्ञात कीजिए । <br> Three balls are drawn one by one without replacement from a bag containing 5 white and 4 green balls . Find the probability distribution of number of green balls drawn. | 2 |
| 6 | 52 पत्तों की एक गड्डी में से यदृच्छया बिना प्रतिस्थापित किए दो पत्ते निकाले गए। एक कार्ड के लाल तथा दूसरे के काले होने की प्रायिकता ज्ञात कीजिए। <br> Two cards are drawn drawn at random one by one without replacement from a well shuffled pack of 52 playing cards . Find the probability that one card red and other is black. | 2 |
| | **खंड – ब** <br> **SECTION – B** | |
| 7 | दिए हुए समाकलन का मान ज्ञात कीजिए । <br> Evaluate the following integral $\int \left( \frac{dx}{1+x+x^2+x^3} \right)$ | 3 |
| 8 | निम्न अवकल समीकरण का व्यापक हल ज्ञात कीजिए.। <br> Find the general solution of the following differential equation: <br> $x\,dy - y\,dx = \sqrt{x^2+y^2}dx$ <br> OR <br> निम्न अवकल समीकरण $\frac{dY}{dx}$ - 3ycotx = sin2x का हल ज्ञात कीजिए.। <br> दिया है y=2 जब $x=\frac{\pi}{2}$ <br> Solve the following differential equation <br> $\frac{dY}{dx}$ - 3ycotx = sin2x ,given that y=2 when $x=\frac{\pi}{2}$ | 3 |
| 9 | $\vec{a},\vec{b},\vec{c}$ तीन सदिश इस प्रकार हैं ताकि $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$ <br> सिद्ध कीजिए ,$\vec{a},\vec{b},\vec{c}$ परस्पर लम्बवत होंगे तथा $\lvert\vec{b}\rvert=1$ ,$\lvert\vec{c}\rvert=\lvert\vec{a}\rvert$ <br> $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$ <br> prove that ,$\vec{a},\vec{b},\vec{c}$ are mutually at right angles and $\lvert\vec{b}\rvert=1$ ,$\lvert\vec{c}\rvert=\lvert\vec{a}\rvert$ | 3 |
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| **10** | दो रेखाओं $\vec{r}=4\hat{i}-3\hat{j}+\lambda(\hat{i}+2\hat{j}-2\hat{k})$ तथा $\vec{r}=\hat{i}+\hat{j}-2\hat{k}-\mu(2\hat{i}+4\hat{j}-4\hat{k})$ के मध्य न्यूनतम दूरी ज्ञात कीजिए।<br>Find the shortest distance between the lines<br><br>$\vec{r}=4\hat{i}-3\hat{j}+\lambda(\hat{i}+2\hat{j}-2\hat{k})$ and $\vec{r}=\hat{i}+\hat{j}-2\hat{k}-\mu(2\hat{i}+4\hat{j}-4\hat{k})$<br>OR<br>समतल का सदिश तथा कार्तीये समीकरण ज्ञात कीजिए जो बिन्दु (1,2,-4) से गुजरता हो तथा $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}=\lambda(2\hat{i}+3\hat{j}+8\hat{k})$ और $\hat{i}-3\hat{j}+5\hat{k}+\mu(\hat{i}+\hat{j}-\hat{k})$के समांतर हो।<br><br>Find the vector and Cartesian forms of the equation of the plane passing through the point (1,2,-4) and parallel to the lines<br>$\vec{r}=\hat{i}+2\hat{j}-4\hat{k}=\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\hat{i}-3\hat{j}+5\hat{k}+\mu(\hat{i}+\hat{j}-\hat{k})$. | 3 |
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| | **खंड – स**<br>**SECTION – C** | |
| **11** | ज्ञात कीजिए<br>Evaluate: $\int_{-1}^{2} \lvert x^3-x \rvert dx$ | 4 |
| **12** | प्रथम चतुर्थांश मे वृत $x^2+y^2=16$ तथा रेखा $\sqrt{3}\,y=x$ से घिरे क्षेत्र का समाकलन विधि द्वारा क्षेत्रफल ज्ञात कीजिए ।<br>Find the area bounded by the circle $x^2+y^2=16$ and the line $\sqrt{3}\,y=x$, in the first quadrant using integration.<br>अथवा / OR<br>वक्र $y=x^3$ , x अक्ष एवं कोटियों x =-2 तथा x =1 से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए<br><br>Find the area bounded by the curve $y=x^3$ , the x axis and ordinate x= -2 and x=1 | 4 |
| **13** | **बिन्दु (1,2,3) से समतल x-y+z=5 की रेखा $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$से समांतर नापी गई दूरी ज्ञात कीजिए ।**<br><br>**F)nd the distance of the point (1,2,3) from the plane x-y+z=5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$.** | 4 |
| **14** | केस आधारित / डेटा आधारित<br>**CASE BASED /DATA BASED**<br><br>कोई भी COVID-19 टेस्ट 100% सही नहीं है, इसलिए उनकी त्रुटियां और परिणाम दोनों महत्वपूर्ण हैं<br>चल रहे **COVID-19** महामारी के दौरान ,पूरी दुनिया मे **RT-PCR** परीक्षणों और तेज एंटीबॉडी -आधारित परीक्षणों दोनों के निदान में संभावित त्रुटियों के बारे मे बहुत चर्चा है ।<br>एक **COVID PCR** परीक्षण की विश्वसनीयता निम्नानुसार निर्दिष्ट की गई है **:-**<br>**COVID** वाले लोगों मे से **, 90%** परीक्षण बीमारी का पता लगते हैं लेकिन **10%** का पता नहीं चल पाता है। **COVID** से मुक्त लोगों मे से **99%** परीक्षण को **COVID** नकारात्मक माना जाता है , लेकिन **1%** को **COVID** सकारात्मक दिखाते हुए निदान किया जाता है । एक बड़ी आबादी मे से केवल **1%** मे ही **COVID** है **,** एक व्यक्ति को याद्दच्छिक रूप से चुना जाता है, जिसे COVID PCR | |

परीक्षण दिया जाता है, और रोगविज्ञानी उसे COVID पॉजिटिव के रूप में रिपोर्ट करता है ।

उपरोक्त जानकारी के आधार पर निम्नलिखित के उत्तर दीजिए:-

(अ ) 'व्यक्ति के COVID पॉजिटिव के रूप में परीक्षण किए जाने की संभावना क्या है' यह देखते हुए कि 'उसे वास्तव में COVID है?
(ब ) इस बात की क्या प्रायिकता है कि 'उस व्यक्ति को वास्तव में कोविड हो गया है, यह देखते हुए कि 'उसकी कोविड पॉजिटिव के रूप में जांच की गई है?

**No COVID-19 Test Is 100% Right, so Their Errors and Results Are Both Important**
During the ongoing COVID-19 pandemic, there is a lot of buzz regarding the possible errors in diagnoses with both RT-PCR tests and the faster antibody-based tests, all over the world.
The reliability of a COVID PCR test is specified as follows:
Of people having COVID, 90% of the test detects the disease but 10% goes undetected.
Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive.

**Based on the above information, answer the following** :-

(a)What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID? **2**
(b).What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive ? **2**

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# Directorate of Education, GNCT of Delhi

## Solution of Practice Paper

## Term -II (2021-22)

## Class – XII

## Mathematics (Code: 041)

| Q. No. | VALUE POINTS |
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| | **SECTION – A** |
| 1 | Let I=$\int\left(\frac{1+x}{1+x^2}\right)dx$=$\int\left(\frac{1}{1+x^2}\right)dx$+$\int\left(\frac{x}{1+x^2}\right)dx$-------(1)<br><br>$\int\left(\frac{x}{1+x^2}\right)dx=\tan^{-1}x$[formula]<br><br>to evaluate $\int\left(\frac{x}{1+x^2}\right)dx$ put 1+$x^2$=t on differentiating both sides we get 2xdx=dt or xdx=$\frac{1}{2}dt$<br><br>$\int\left(\frac{x}{1+x^2}\right)dx$=$\int\left(\frac{1}{2}\frac{dt}{t}\right)$=$\frac{1}{2}$log\|t\|=$\frac{1}{2}$log\|$1+x^2$<br><br>=$\frac{1}{2}$log$1+x^2$<br><br>Putting these values in equation( 1 )we have I=$\tan^{-1}x$+$\frac{1}{2}$log$1+x^2$+c<br><br>OR<br><br>SOL-$\int\left(\frac{3x-6+6-1}{(x-2)^2}\right)dx$=$\int\left(\frac{3(x-2)+5}{(x-2)^2}\right)dx$=3log\|x-2\|\|+5$\int(x-2)^{-2}dx$<br><br>=3log\|x-2\|+5$\frac{(x-2)^{-1}}{-1}$+c=3log\|x-2\|-$\frac{5}{x-2}$+c |
| 2 | Given differential equation is<br>$\frac{d}{dx}\{\left(\frac{dy}{dx}\right)^3\}=0$<br>=>$3\left(\frac{dy}{dx}\right)^{3-1}\frac{d}{dx}\left(\frac{dy}{dx}\right)$=0<br>=>$3\left(\frac{dy}{dx}\right)^2\frac{d^2y}{dx^2}$=0<br>clearly the highest order derivative occurring in the differential equation is $\frac{d^2y}{dx^2}$so its order is 2. Also it is a polynomial equation in derivative and the highest power raised to is $\frac{d^2y}{dx^2}$one so its degree is one . Hence the sum of the order and degree of the above given differential equation is 2+1=3 |

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| 3 | Sol-Let $\hat{a}$and $\hat{b}$be unit vectors such that $\hat{a}+\hat{b}$is also a unit vector<br>$\|\hat{a}\|=\|\hat{b}\|=\|\hat{a}+\hat{b}\|$----------------(1)<br>we know that $\left\|\hat{a}+\hat{b}\right\|^2=(\hat{a}+\hat{b})^2=(\hat{a})^2+(\hat{b})^2+2\,\hat{a}\,\hat{b}$-----------(2)<br>$\left\|\hat{a}-\hat{b}\right\|^2=(\hat{a}-\hat{b})^2=-(\hat{a})^2+(\hat{b})^2-2\,\hat{a}\,\hat{b}$-------(3)<br>adding equations (2) and (3) we have<br>$\left\|\hat{a}+\hat{b}\right\|^2+\left\|\hat{a}-\hat{b}\right\|^2=2(\hat{a})^2+2(\hat{b})^2=\left\|\hat{a}\right\|^2+2\left\|\hat{b}\right\|^2$<br>putting the values of $\left\|\hat{a}\right\|\left\|\hat{b}\right\|\left\|\hat{a}+\hat{b}\right\|$(each=1)<br>$1^2+\left\|\hat{a}-\hat{b}\right\|^2=2\left\|\hat{a}\right\|^2+2\left\|\hat{b}\right\|^2=2(1)+2(1)$<br>$\left\|\hat{a}-\hat{b}\right\|^2=3 \Rightarrow \left\|\hat{a}-\hat{b}\right\|=\sqrt{3}$ |
| 4 | $\frac{8}{\sqrt{89}},\frac{-4}{\sqrt{89}},\frac{3}{\sqrt{89}}$ |
| 5 | Sol-<br>Three balls are drawn one by one without replacement from a bag containing 5 whit and 4 green balls .<br>Let X denote number of green balls (out of three green balls drawn)<br>=> X=0,1,2 and 3 only(and not upto 4 , the number of green balls in the bag<br>P(X=0)= Probability of getting green no green balls in the three draws i.e., all the three white balls =P(WWW)=$\frac{5}{9}x\frac{4}{8}x\frac{3}{7}=\frac{60}{504}=\frac{5}{42}$<br>P(X=1)= Probability of getting one green balls (and hence two white balls ) in three draws<br>=P(GWW)+P(WGW)+P(WWG)=$\frac{4}{9}x\frac{5}{8}x\frac{4}{7}+\frac{5}{9}x\frac{4}{8}x\frac{4}{7}+\frac{5}{9}x\frac{4}{8}x\frac{4}{7}$<br>$=\frac{240}{504}=\frac{20}{42}$<br>similarly P(X=2)= Probability of getting two green balls (and hence one white ball ) in three draws=$\frac{15}{42}$<br>and similarly P(X=3)= Probability of getting all the three green balls =<br>P(GGG)=$\frac{4}{9}x\frac{3}{8}x\frac{2}{7}=\frac{2}{42}$<br>Hence probability distribution of X is<br><table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X)</td><td>$\frac{5}{42}$</td><td>$\frac{20}{42}$</td><td>$\frac{15}{42}$</td><td>$\frac{15}{42}$</td></tr></table> |
| 6 | Solution- There are 26 red cards and 26 black cards in a pack of 52 playing cards .<br>Let R and B denote the event of drawing red card and black card respectively<br>Now required probability=P(first card is red and second one is black)+P(first card is black and second one is red)<br>=P(R)+$P\left(\frac{B}{R}\right)$+P(B)$P\left(\frac{R}{B}\right)$<br>$=\frac{26}{52}x\frac{26}{51}+\frac{26}{52}x\frac{26}{51}$<br>$=\frac{26}{51}$ |
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| | **SECTION – B** |
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| 7 | Let I=$\int\left(\frac{dx}{1+x+x^2+x^3}\right)=\int\left(\frac{dx}{(1+x)+x^2(1+x)}\right)=\int\left(\frac{dx}{(1+x)(1+x^2)}\right)$<br>let$\left(\frac{dx}{(1+x)(1+x^2)}\right)=\frac{A}{1+X}+\frac{BX+C}{1+x^2}$--------------(1)<br>Multiply both sides by L.C.M $(1+x)(1+x^2)$we get<br>1= A$(1+x^2)$+(BX+C) $(1+x)$<br>OR 1=A+A$x^2$+Bx+B $x^2$+C+Cx<br>Equating coefficients of $x^2$,x and constant term ,we get<br>A+B=0<br>B+C=0<br>Putting values<br>A+C=1<br>Solving A,B,C we get A=$\frac{1}{2}$,C=$\frac{1}{2}$,B=-A=$\frac{-1}{2}$<br>Putting values of A,B,C in (1) we get<br>$\left(\frac{1}{(1+x)(1+x^2)}\right)=\frac{\frac{1}{2}}{1+X}+\frac{\left(\frac{-1}{2}x+\frac{1}{2}\right)}{1+X^2}$<br>$=\frac{1}{2}\frac{1}{1+X}-\frac{1}{2}\frac{x}{1+x^2}+\frac{1}{2}\frac{1}{1+x^2}$<br>Hence I=$\frac{1}{2}\log\lvert(1+x)\rvert-\frac{1}{4}\log(1+x^2)+\frac{1}{2}\tan^{-1}x$+c |
| 8 | The given differential equation is<br>$xd\,y$ −ydx= $\sqrt{x^2+y^2}$dx<br>dividing by dx<br>x$\frac{dy}{dx}$-y=<br>=>x$\frac{dy}{dx}$=y+$\sqrt{x^2+y^2}$=>$\frac{dy}{dx}$-$\frac{y}{x}$=$\frac{\sqrt{x^2+y^2}}{x}$<br>Put y=vx=>$\frac{dy}{dx}$=v+x$\frac{dv}{dx}$<br>Therefore<br>v+x$\frac{dv}{dx}$-$\frac{vx}{x}$=$\frac{\sqrt{x^2+v^2x^2}}{x}$=>v+x$\frac{dv}{dx}$-v=$\sqrt{1+v^2}$<br>=>$\int\left(\frac{dv}{\sqrt{1+v^2}}\right)=\int\left(\frac{dx}{x}\right)$=>log \|v+$\sqrt{1+v^2}$\|=log\|x\|+log\|c\|=>loglog$\left\lvert\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right\rvert$=log\|Cx\|=>$\frac{y}{x}$+$\sqrt{\left(\frac{x^2+y^2x^2}{x^2}\right)}$<br>=Cx=>y+$\sqrt{x^2+y^2}$=C$x^2$<br>**OR**<br>The given differential equation is<br>$\frac{dY}{dx}$-3ycotx=sin2x given that y=2 when x=$\frac{\pi}{2}$<br>Compairing with $\frac{dY}{dx}$+Py=Q we have P=3cotx and Q=sin2x |

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|---|---|
| | $\int Pdx$<br>$=-3\int cotxdx=-3\log sinx=\log(sinx)^{-3}$<br>IF$==\int e^{pdx}=e^{\log(sinx)^{-3}}=(sinx)^{-3}=\frac{1}{\sin^3 x}$<br>The general solution is y(IF.)$=\int Q(IF)$dx+C<br>or $y\frac{1}{\sin^3 x}=\int \sin 2x\frac{1}{\sin^3 x}$dx+C<br>$\frac{y}{\sin^3 x}=\int \frac{(2 sinxcosx)}{\sin^3 x}$dx+C<br>$=2\int \frac{cosx}{\sin^2 x}$dx+C$=2\int \frac{cosx}{sinxcosx}$dx+C$=\int 2 cosecx\, cotx$dx=-2cosecx+C<br>or<br>$\frac{y}{\sin^3 x}=\frac{-2}{sinx}$+C<br>$y=\sin^3 x$<br>multiplying every term by LCM$=\sin^3 x$<br>$y=-2\sin^2 x+c\sin^3 x$To find C putting y=2 when $x=\frac{\pi}{2}$(given in (1)<br>$2=-2\sin^2\frac{\pi}{2}+c\sin^3\frac{\pi}{2}$<br>or 2=-2+c or c=4 puttung c=4 the required particular solution is<br>$y=-2\sin^2 x+4\sin^3 x$ |
| 9 | $\vec{a}X\vec{b}=\vec{c}\Rightarrow\vec{c}\perp\vec{a}$and$\vec{c}\perp\vec{b}$<br>(By def of cross product) ----------(1)<br>similarly $\vec{b}\,x\,\vec{c}=\vec{a}\Rightarrow\vec{a}\perp\vec{b}$and $\vec{a}\perp\vec{c}$-----------(2)<br>from (1) and (2) we have$\vec{a},\vec{b},\vec{c}$are mutually at right angles.<br>Now $\vec{a}\,x\,\vec{b}=\vec{c}$(given)$\Rightarrow\lvert\vec{a}\,x\,\vec{b}\rvert=\lvert\vec{c}\rvert$<br>Using (2)$\lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin 90^0=\lvert\vec{c}\rvert$<br>$\lvert\vec{a}\rvert\lvert\vec{b}\rvert=\lvert\vec{c}\rvert$-------(3)<br>similarly$\vec{b}\,x\,\vec{c}=\vec{a}\Rightarrow\lvert\vec{b}\,x\,\vec{c}\rvert=\lvert\vec{a}\rvert$<br>$\Rightarrow\lvert\vec{b}\rvert\lvert\vec{c}\rvert=\lvert\vec{a}\rvert$----(4)<br>Dividing (3) by(4) ( to eliminate $\lvert\vec{b}\rvert$) we have<br>$\frac{\lvert\vec{a}\rvert}{\lvert\vec{c}\rvert}=\frac{\lvert\vec{c}\rvert}{\lvert\vec{a}\rvert}\Rightarrow\lvert\vec{a}\rvert^2=\lvert\vec{c}\rvert^2$therefore $\lvert\vec{a}\rvert=\lvert\vec{c}\rvert$------(5)<br>dividing (3) by (5) $\lvert\vec{b}\rvert$=1 putting it in (4) we have<br>$\lvert\vec{c}\rvert=\lvert\vec{a}\rvert$ |
| 10 | Solution-Here $\vec{b_1}=\hat{i}+2\hat{j}-2\hat{k}$and $\vec{b_2}=2\hat{i}+4\hat{j}-4\hat{k}=2(\hat{i}+2\hat{j}-2\hat{k})$<br>$=2\vec{b_1}$Therefore , the lines are parallel now using the formula for distance between parallel lines = $\frac{\lvert(\vec{a_2}-\vec{a_1})\times\vec{b}\rvert}{\lvert\vec{b}\rvert}$<br>Shortest distance is $2\sqrt{5}$<br>**OR**<br>Sol-<br>Given point is $\vec{a_1}$=(1 ,2 ,-4 ) $=\hat{i}+2\hat{j}-4\hat{k}$ we know that vector along the line $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}=\lambda(2\hat{i}+3\hat{j}+6\hat{k})$is $\vec{b_1}=(2\hat{i}+3\hat{j}+6\hat{k})$and a vector along the line $\vec{b_2}=(\hat{i}+\hat{j}-\hat{k})$<br>we know vector equation of the plane passing through point $\vec{a}$and parallel to the two given lines is ($\vec{r}$ -$\vec{a_1}$).$\vec{n}$=0<br>where $\vec{n}=\vec{b_1}$x$\vec{b_2}$-------------------(1)<br>now $\vec{n}=\vec{b_1}$x$\vec{b_2}$ |

$=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$

$=\hat{i}(-3-6)-\hat{j}(-2-6)+\hat{k}(2-3)=-9\hat{i}+8\hat{j}-\hat{k}$

from equation (1)$\vec{r}.\vec{n}-\vec{a}_1.\vec{n}=0 \Rightarrow \vec{r}.\vec{n}=\vec{a}_1.\vec{n}$-------(2)

putting values of $\vec{a}_1$ and $\vec{n}$ in (2) equatoin of required plane is

i.e.,$\vec{r}.(-9\hat{i}+8\hat{j}-\hat{k})=(\hat{i}+2\hat{j}-4\hat{k}).(-9\hat{i}+8\hat{j}-\hat{k})=-9+16+4$

or $\vec{r}.(-9\hat{i}+8\hat{j}-\hat{k})=11$ **(Vector form of the plane )**

or$(x\hat{i}+y\hat{j}+z\hat{k}).(-9\hat{i}+8\hat{j}-\hat{k})=11$

or 9x+8y-z=11

or 9x+8y-z-11=0 **(Cartesian form of the plane)**

## SECTION – C

**11** Let $I=\int_{-1}^{2}|x^3-x|dx$

Again let f(x) $=x^3-x=x(x^2-1)=x(x-1)(x+1)$

Now break the limit at x=0,1 (because on putting f(x)=0 we get x=0,1,-1)

It is clear that $x^3-x\geqslant 0$ on[-1,0]

$x^3-x\leqslant 0$ on[0,1]

$x^3-x\geqslant 0$ on[1,2]

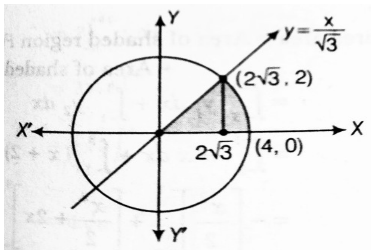
Hence the interval of the integral can be subdivided as

$\int_{-1}^{2}|x^3-x|dx=\int_{-1}^{0}(x^3-x)dx-\int_{0}^{1}(x^3-x)dx+\int_{1}^{2}(x^3-x)dx$

$==\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}(x-x^3)dx+\int_{1}^{2}(x^3-x)dx=\frac{11}{4}$

**12** Given equation of the circle is $x^2+y^2=16$ and $\sqrt{3}y=x$, represents a line through origin.

The line $y=\frac{1}{\sqrt{3}}x$ intersect the circle .




Therefore $x^2+\frac{x^2}{3}=16$

$\frac{3x^2+x^2}{3}=16 \Rightarrow 4x^2=48$

$\Rightarrow x^2=12 \Rightarrow x=\pm 2\sqrt{3}$

When $x=2\sqrt{3}$ then $y=\frac{2\sqrt{3}}{\sqrt{3}}=2$

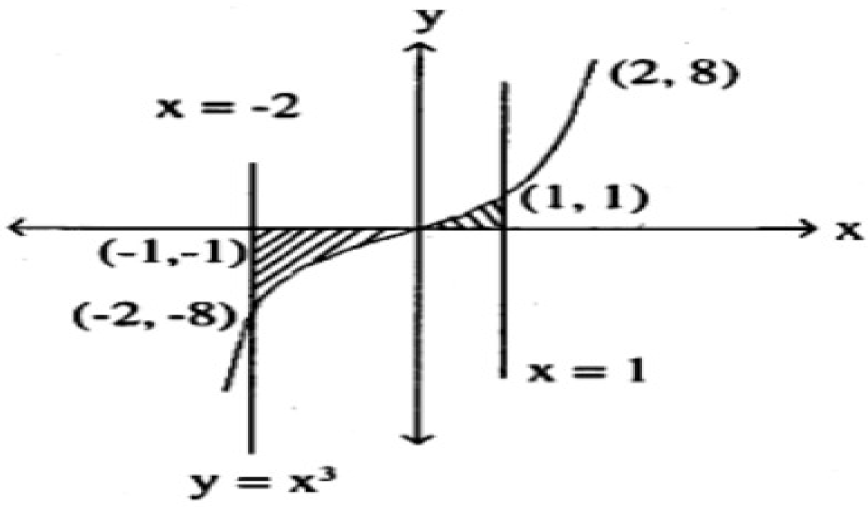
Required area shaded in first quadrant =(Area under the line $y=\frac{1}{\sqrt{3}}x$ from x=0 to $2\sqrt{3}$) +

( Area under the circle from $x=2\sqrt{3}$ to x=4

$=\int_{0}^{2\sqrt{3}}\frac{1}{\sqrt{3}}x\,dx+\int_{2\sqrt{3}}^{4}\sqrt{16-x^2}\,dx$

after solving it we get required area $=\frac{4\pi}{3}$ sq units

OR




Required area (shaded)

= Area under curve y = $x^3$ with respect to 'x' axis from x = -2 to x = 1

$$= \int_{-2}^{1}|x^3|\,dx = \left|\int_{2}^{0}|x^3|dx\right| + \int_0^1|x^3|dx$$

$$= \left|\int_{-2}^{0}-x^3dx\right| + \int_0^1 x^3dx$$

As cube a value below 0 is negative

$$= \left.\frac{-x^4}{4}\right]_{-2}^{0} + \left.\frac{x^4}{4}\right]_0^1 = -\left[0-\frac{(-2)^4}{4}\right]+\left[\frac{1}{4}-\frac{0}{4}\right]$$

$$= -\left[\frac{-16}{4}\right]+\frac{1}{4} = \frac{16}{4}+\frac{1}{4} = \frac{17}{4} \text{ sq.units}$$

**13**

We know that d.r.'s of the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ are is denominators 2 ,3 ,-6

therefore d.r.'s of any line parallel to it are also 2 ,3 ,-6 therefore equation of the line through P (1 ,-2 ,3 ) and parallel to the given line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ are

$\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ $\qquad \frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}(=\lambda)$--------(1)

let this line meet the given plane x-y+z=5 in the point Q say ) from equation (1) x-1=2$\lambda$ y+2=3 $\lambda$,z-3=-6$\lambda$=>x=2$\lambda$+1 ,y=3$\lambda$-2

z=-6$\lambda$+3

therefore point Q is Q( 2$\lambda$+1 , 3$\lambda$-2 , -6$\lambda$+3 ) for some real $\lambda$

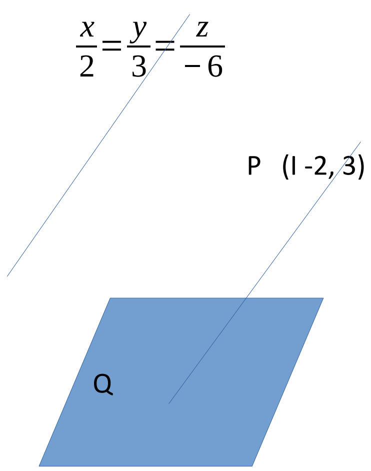
But this point Q lies on the plane x-y+z=5 therefore

(2$\lambda$+1)-(3$\lambda$-2)(-6$\lambda$+3 )=

2$\lambda$+1-3$\lambda$+2-6$\lambda$+3 =5=>-7$\lambda$=-1=>$\lambda=\frac{1}{7}$

putting $\lambda=\frac{1}{7}$ in (3) coordinates of point Q are $(\frac{9}{7}, \frac{-11}{7}\ \frac{15}{7})$

Required distance is PQ




$$\sqrt{\left(\frac{9}{7}-1\right)^2+\left(\frac{-11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2}=\mathbf{1}$$

$$\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{-6}{7}\right)^2}$$

| | |
|---|---|
| | $\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=\sqrt{\frac{49}{49}}=\mathbf{1}$ |
| **14** | (a)0.9 2.<br>(b)0.083 |